Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Higher level
Paper 1

14 May 2026

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Session number

2 hours

Instructions to students

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

2226 – 5020

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

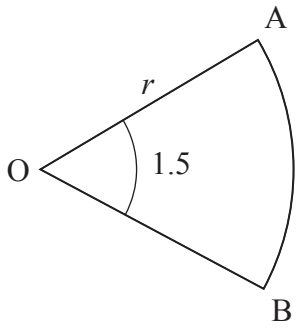
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

A sector AOB of radius $r\,\mathrm{cm}$, where $\mathrm{A\hat{O}B} = 1.5$ radians, is shown in the following diagram.

diagram not to scale



(a) (i) Find an expression for the length of arc AB in terms of r.

(ii) Hence, find an expression for the perimeter of sector AOB in terms of r. [2]

(b) Given that the perimeter of the sector AOB is $14\,\mathrm{cm}$, find the value of r. [2]

(c) Hence, find the area of sector AOB. [2]

(This question continues on the following page)

(Question 1 continued)

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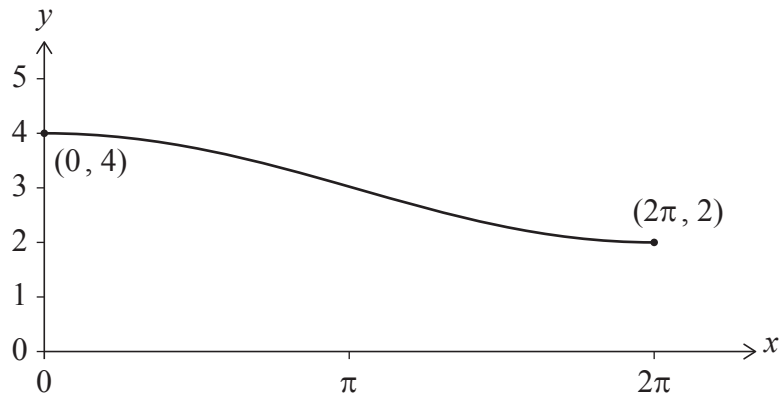
2. [Maximum mark: 6]

Consider the function $f(x) = 3 + \cos bx$ for $0 \leq x \leq 2\pi$, where $b \in \mathbb{R}^+$.

The graph of $y = f(x)$ is shown in the following diagram.



The points $(0, 4)$ and $(2\pi, 2)$ are the maximum and minimum points, respectively, on the graph of $y = f(x)$.

(a) Show that $b = \frac{1}{2}$. [1]

(b) Find the area enclosed by the graph of $y = f(x)$, the coordinate axes and the line $x = 2\pi$. [5]

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3. [Maximum mark: 6]

The functions f and g are defined by

$$f(x) = x^2 + 5x,\ x \in \mathbb{R}$$

$$g(x) = e^x - 10,\ x \in \mathbb{R}.$$

Find the value of x for which $(f \circ g)(x) = 50$.

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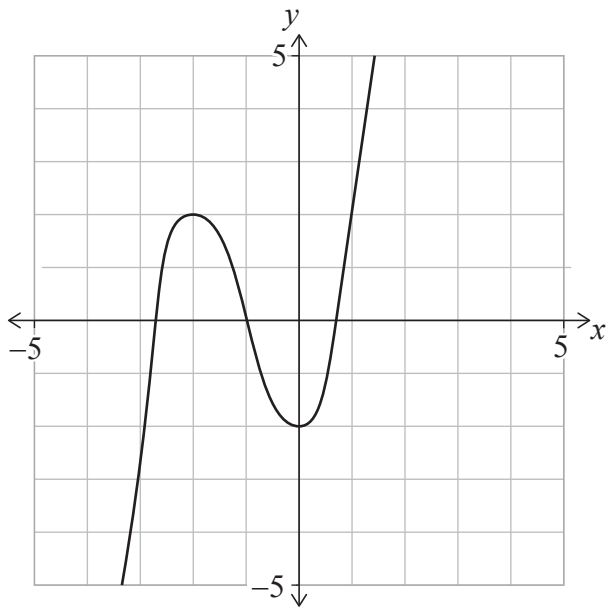
4. [Maximum mark: 6]

Consider the function $f(x) = x^3 + 3x^2 - 2$, where $x \in \mathbb{R}$.

The following diagram shows part of the curve $y = f(x)$.



One root of the equation $f(x) = 0$ is $x = -1$.

(a) Find the values of the other two roots of the equation $f(x) = 0$. [4]

(b) Hence, write down the three roots of the equation $f\left(\frac{x}{2}\right) = 0$. [2]

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5. [Maximum mark: 6]

Consider

$$\sum_{n=2}^{8} \ln\left(1-\frac{1}{n}\right) = \ln\left(1-\frac{1}{2}\right)+\ln\left(1-\frac{1}{3}\right)+\ldots+\ln\left(1-\frac{1}{8}\right), \text{ where } n \in \mathbb{Z}^+.$$

(a) Show that $\sum_{n=2}^{8} \ln\left(1-\frac{1}{n}\right) = -\ln a$, where a is a constant to be determined, $a \in \mathbb{Z}^+$. [5]

(b) Hence, write down the value of $\sum_{n=2}^{2026} \ln\left(1-\frac{1}{n}\right)$. [1]

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6. [Maximum mark: 5]

A committee of 3 teachers and 2 students is to be selected from 5 teachers and 4 students.

Find the number of possible committees that can be formed, if 2 particular students cannot be on the committee together.

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7. [Maximum mark: 7]

Prove by mathematical induction that $n^3 + 5n$ is a multiple of 6 for $n \in \mathbb{Z}^+$.

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8. [Maximum mark: 7]

A curve C is given by $2y + 3xy^2 - 2x^2 + 4 = 0$, for $x \geq \sqrt{2}$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. [4]

The point $\mathrm{P}(2, -1)$ lies on C. A particle travels along C such that, at P, the instantaneous rate of change of the y-coordinate with respect to time is -6 units per second.

(b) Find the rate of change of the x-coordinate with respect to time at P. [3]

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9. [Maximum mark: 7]

Solve the differential equation $xy\frac{\mathrm{d}y}{\mathrm{d}x} = 1 + y^2$, given that $y = 0$ when $x = 2$.

Give your answer in the form $y^2 = f(x)$.

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Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 15]

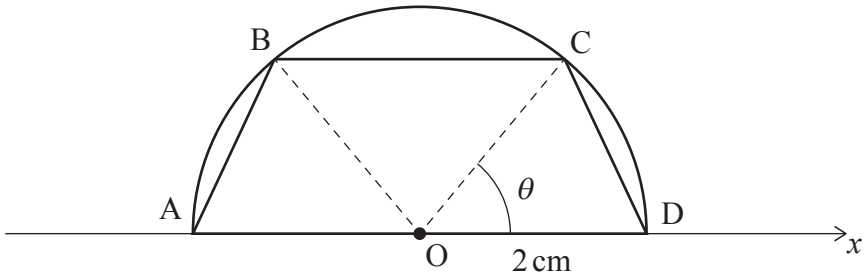
A trapezoid ABCD is inscribed in a semicircle centred at O, of radius $2\,\mathrm{cm}$.

The longer parallel side $[\mathrm{AD}]$ is along the diameter of the semicircle.

The trapezoid is defined by the angle, θ, when the radius is rotated anticlockwise (counterclockwise) from the positive x-axis where $0 < \theta < \frac{\pi}{2}$.

This is shown in the following diagram.

diagram not to scale



The area of the trapezoid can be considered as the sum of the area of three triangles.

(a) Find the area of the trapezoid when $\theta = \frac{\pi}{6}$. [4]

(b) Find an expression for the area of the trapezoid $A(\theta)$, in terms of θ. Give your answer in the form $a\sin\theta + b\sin 2\theta$, where $a, b \in \mathbb{Z}^+$. [3]

(c) (i) Find $A'(\theta)$.

(ii) Hence, find the maximum possible area of the trapezoid. [6]

Consider the case when θ approaches $\frac{\pi}{2}$.

(d) (i) Describe the effect on the shape of ABCD.

(ii) The area of ABCD approaches the value k. Find the value of k. [2]

Do **not** write solutions on this page.

11. [Maximum mark: 17]

Consider the function $f(x) = \frac{-x-7}{x^2+4x+3}$, where $x \in \mathbb{R}, x \neq -3, x \neq -1$.

(a) Use partial fractions to show that $f(x)$ can be expressed as $\frac{2}{3+x} - \frac{3}{1+x}$. [3]

(b) Show that the area of the region enclosed by the graph of $y = f(x)$, the x-axis and the lines $x = 0$ and $x = 1$ is $\ln\frac{9}{2}$. [5]

(c) Show that the binomial expansion of $f(x)$ up to and including the x^2 term is $-\frac{7}{3} + \frac{25}{9}x - \frac{79}{27}x^2$. [4]

(d) (i) Use the result from part (c) to find an approximation of $\int_0^1 f(x)\,\mathrm{d}x$.

(ii) Hence, write down the area enclosed by the x-axis, the graph of $y = -\frac{7}{3} + \frac{25}{9}x - \frac{79}{27}x^2$ and the lines $x = 0$ and $x = 1$. [3]

The value of $\ln\frac{9}{2}$ correct to three significant figures is 1.50.

(e) Determine whether the binomial expansion approximation is an underestimate or an overestimate of the area found in part (b). Justify your answer. [2]

Do **not** write solutions on this page.

12. [Maximum mark: 22]

Consider the quadratic polynomial $Q(z) = z^2 - 2\sqrt{3}z + 4$, where $z \in \mathbb{C}$.

(a) Find the roots of $Q(z) = 0$,

(i) in Cartesian form;

(ii) in modulus-argument form. [6]

Now consider the polynomial $P(z) = z^6 + 64$, where $z \in \mathbb{C}$.

(b) Solve $P(z) = 0$. Give your answers in modulus-argument form. [5]

(c) Plot the roots of the equation $P(z) = 0$ on an Argand diagram. Clearly label each root $z_1, z_2, \ldots, z_6$, where z_2 is the root located on the positive imaginary axis. [2]

(d) Verify that $z^2 + 4$ is a factor of $P(z)$. [2]

The polynomial $P(z)$ can be written as the product of three quadratic factors with real coefficients.

(e) Find the other two quadratic factors of $P(z)$. [4]

Line segments are drawn from z_2 to each of the other roots.

(f) Find the sum of the lengths of these line segments. [3]

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Please **do not** write on this page.

Answers written on this page
will not be marked.